Total No. of printed pages = 5

MATHEMATICS – II

Full Marks – 70

Pass Marks – 21

Time – Three hours

The figures in the margin indicate full marks for the questions.

GROUP – A

1. (a) If $f(x) = 1 + e^x$, find $f(f(x))$. 2

(b) Find the domain of the function $f(x) = \sqrt{x^2 - 1}$. 2

(c) Examine the continuity of $f(x)$ where

$$f(x) = \frac{|x-1|}{x-1} \quad \text{if } x \neq 1$$

$$= 0 \quad \text{if } x = 1$$

at $x = 1$. 3

2. Find the limit (any *two*) : 2×2=4

(a) $\underset{x\to 9}{\text{Lt}} \dfrac{\sqrt{x}-3}{x-9}$

(b) $\underset{x\to 0}{\text{Lt}} \dfrac{\tan \frac{x}{3}}{x}$

(c) $\underset{x\to \infty}{\text{Lt}} \dfrac{4x^2-5x+1}{5x^2+2x+3}$

3. Find $\dfrac{dy}{dx}$ (any *three*) : 3×3=9

(a) $y = e^{f(x)}$

(b) $y = \dfrac{e^x}{2+x}$

(c) $x^y = y^x$

(d) $x = a(t + \sin t)$, $y = b \cos t$

4. Find $\dfrac{d^2y}{dx^2}$ (any *two*) : 2×3=6

(a) $y = e^x \tan x$

(b) $y = \cos^{-1} x$

(c) $y = \sin^5 x \cos x$

5. Find the equation of the tangent to the curve $\sqrt{x} + \sqrt{y} = 3$ at (4, 1). 3

6. Find the extreme values of the function $f(x) = 2x^3 - 9x^2 + 12x + 5$. 3

GROUP – B

7. Integrate any *three* : 3×2=6

(a) $\int (\cos x)^2 \, dx$

(b) $\int \left(x^2 + \frac{1}{x^2} \right)^3 dx$

(c) $\int x^2 \log x \, dx$

(d) $\int \frac{\cos x}{1 + \sin^2 x} dx$

8. Evaluate any *two* : 3×2=6

(a) $\int_0^1 xe^x \, dx$

(b) $\int_0^1 \frac{\tan^{-1} x}{1 + x^2} dx$

(c) $\int_0^{\frac{\pi}{2}} \frac{\sin x}{\sin x + \cos x} dx$

9. Find by the method of integration the area of the region bounded by the parabola $y^2 = 8x$ and its latus rectum. 3

10. Find the sum : 3

$$\lim_{n \to \infty} n\left[\frac{1}{n^2 + 1^2} + \frac{1}{n^2 + 2^2} + \ldots\ldots + \frac{1}{n^2 + n^2}\right]$$

GROUP – C

11. Answer any *seven* questions : 7×2=14

(a) Find the centroid of the triangle with vertices (0, 0), (2, 4), (4, 0).

(b) Show that the points (4, 4), (6, 2) and (7, 1) are collinear.

(c) Find the equation of the straight line parallel to $x = 2y$ and passing through (1, 1).

(d) Find intercepts on axes by the straight line $2x + 3y - 5 = 0$.

(e) What is the equation of directrix of the parabola $y^2 = 16x$? M

(f) Express $\frac{x}{2} + \frac{y}{3} = 1$ in perpendicular form.

(g) Write down the equation of tangent to the circle $x^2 + y^2 = a^2$ at (x_1, y_1).

(h) What are the lengths of major axis and minor axis of the ellipse $9x^2 + 16y^2 = 144$.

12. Find the equation of circle passing through the set of points (0, 0), (a, 0) and (0, b). 3

13. Find the co-ordinates of the centre, vertices, focii and the equation of the directrices of the hyperbola $9x^2 - 16y^2 = 144$. 3